Low-lying spectra of ${}^{9}_{\Lambda}$Be and ^{9}Be within three-cluster model

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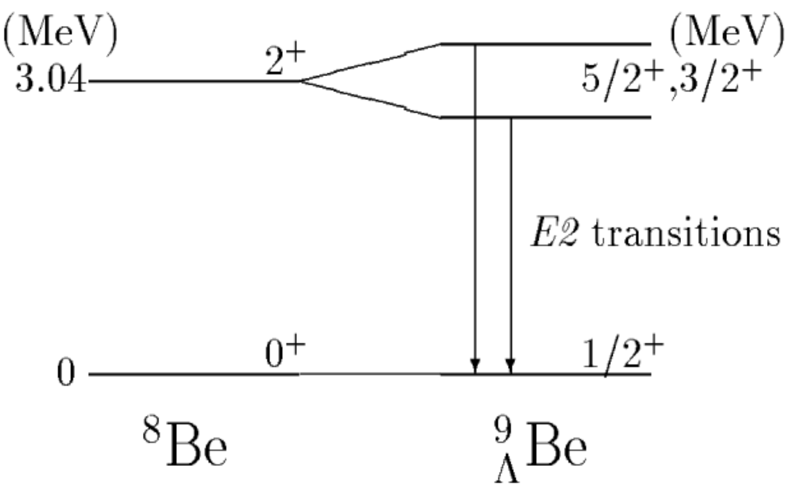
Study of the spin-orbit components of hyperon-nucleon potential is important to investigate the nature of the short-range baryon-baryon interaction.

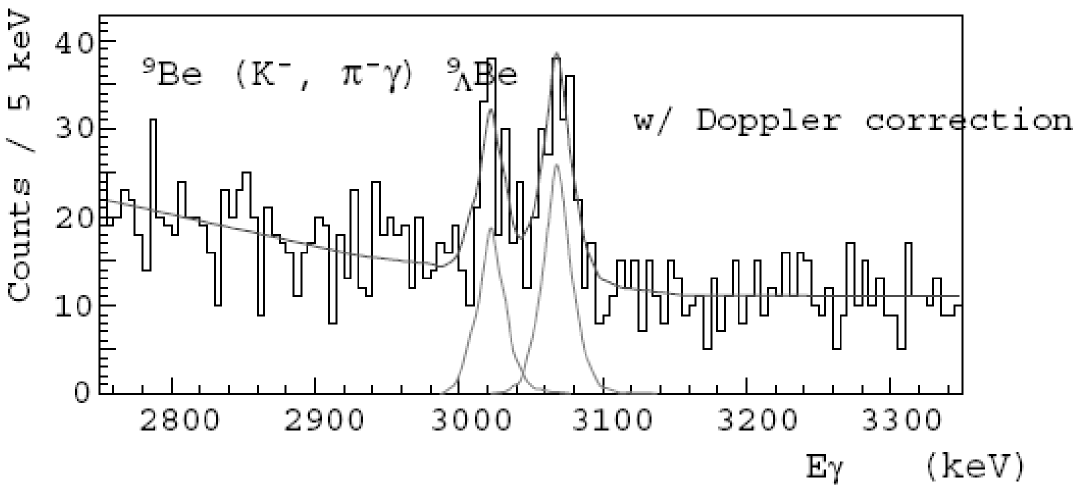
ΛN *spin-orbit potential vary significantly* in different models of the baryon-baryon interaction.

OBE models and quark models have different predictions for spin-orbit splitting of L single-particle states.

$^{9}_{\Lambda}$Be (3/2+, 5/2+) spin-orbit splitting



H. Akikawa et al., Phys. Rev. Lett. 88, 082501 (2002);



Doppler-shift corrected spectrum of $^9_\Lambda$Be γ rays around 3 MeV obtained in the E930('98) experiment.

H. Tamura et al., Nucl. Phys. A754 58c (2005).

$$\Delta E_{so} = 43 \pm 5 \text{ keV}$$

P-shell hypernuclei

$$^{7}_{\Lambda}\mathrm{Li},\ \ ^{10}_{\Lambda}\mathrm{B},\ \ ^{11}_{\Lambda}\mathrm{B},\ \text{and}\ \ ^{16}_{\Lambda}\mathrm{O}$$

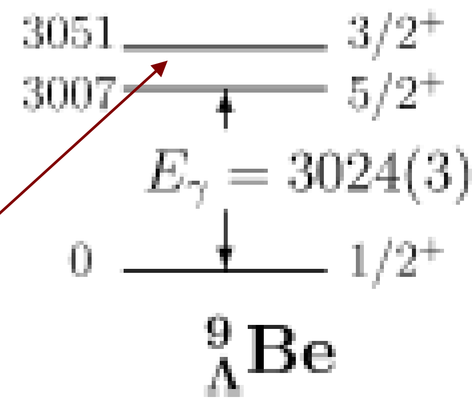
P-shell hypernuclei : methods

Shell model interpretation with effective potential

$$V_{\Lambda N}^{eff}(r) = V_0(r) + V_\sigma(r)\boldsymbol{\sigma}_\Lambda\boldsymbol{\sigma}_N + V_\Lambda(r)\mathbf{l}_{\Lambda N}\boldsymbol{\sigma}_\Lambda + V_N(r)\mathbf{l}_{\Lambda N}\boldsymbol{\sigma}_N + V_T(r)\left[3(\boldsymbol{\sigma}_\Lambda\hat{\mathbf{r}})(\boldsymbol{\sigma}_N\hat{\mathbf{r}}) - \boldsymbol{\sigma}_\Lambda\boldsymbol{\sigma}_N\right]$$

The radial integrals of V_σ, V_Λ, V_N, and V_T with the $p_N s_\Lambda$ wavefunction in p-shell hypernuclei are denoted as Δ, S_Λ, S_N, and T, respectively.

R.H. Dalitz and A. Gal, Ann. Phys. 116 (1978) 167.
D.J. Millener, A. Gal, C.B. Dover and R.H. Dalitz, Phys. Rev. C 31 (1985) 499.
D.J. Millener, Nuclear Physics A 804 (2008) 84–98



3/2⁺ is upper member

$3/2^+ - 5/2^+$ separation

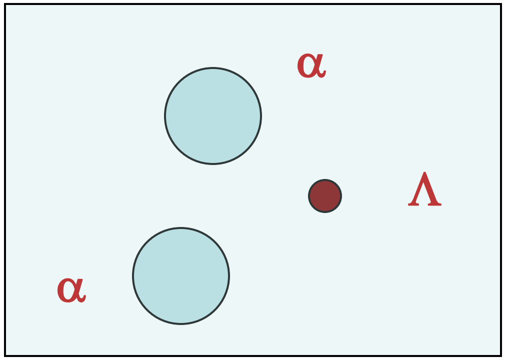
Cluster model interpretation

T. Yamada, K. Ikeda, H. Bando, Prog. Theor. Phys. 73, 397 (1985).
Y. Fujiwara, M. Kohno, K. Miyagawa, Y. Suzuki, Phys. Rev. C70 (2004) 047002
I. Filikhin, V.M. Suslov and B. Vlahovic, Nuclear Physics A 790, 695 (2007)

Variational Monte Carlo calculations (nine particles)

M. Shoeb and Sonika, PRC 79, 054321 (2009)

Cluster model for ${}^{9}_{\Lambda}\mathbf{Be}$

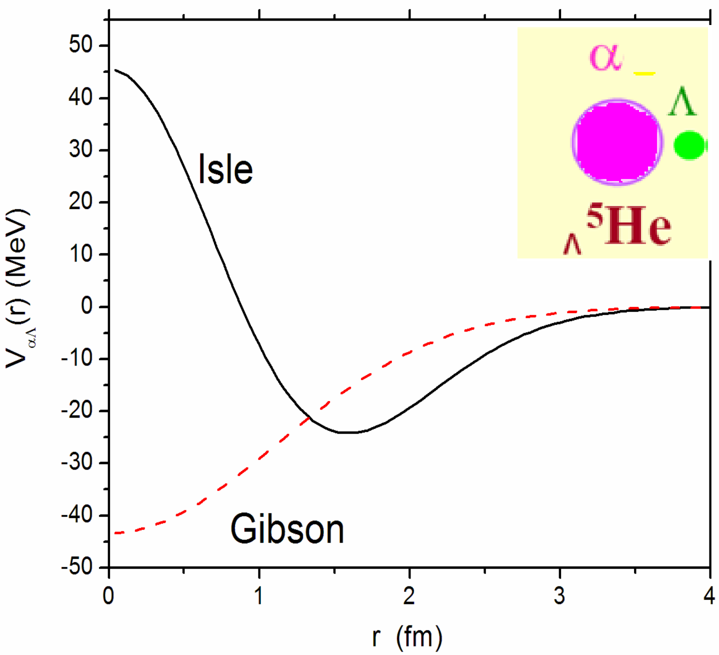


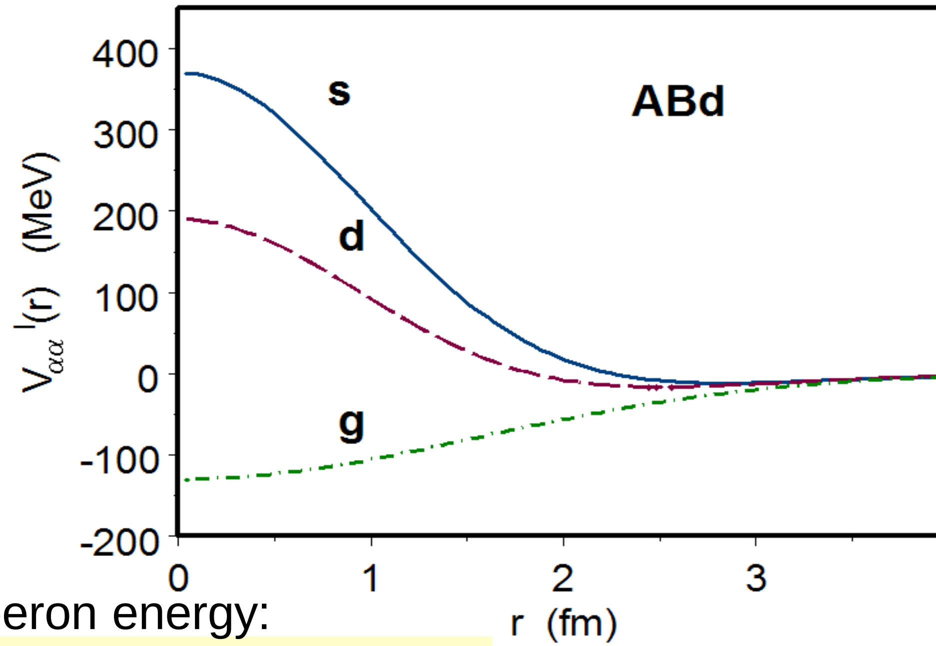
αα–potential

S. Ali, A. R. Bodmer Nucl. Phys. 80 (1966) 99

$$V^l_{\alpha\alpha}(r) = V^l_{rep}\exp(-r/\beta^l_{rep})^2 - V^l_{att}\exp(-r/\beta^l_{att})^2 \quad ,$$

αΛ–potential





Λ hyperon energy:

$$B_\Lambda({}_{\Lambda}{}^{5}\mathrm{He}) = 3.12 \pm 0.02\ \mathrm{MeV}$$

Gibson, Tang – Herndon (**TH**) potentials have a form of one range Gaussian

$$V(x) = V_o \exp(-\alpha x^2)$$

--- Y. Kurihara, Y. Akaishi, H. Tanaka, Phys. Rev. C 84 (1985) 971.

- - - C. Daskaloyannis, M. Grypeos, H. Nassena, Phys. Rev. C 26 (1982) 702.

Formalism

The Faddeev equations in configuration space

$$\boxed{\Psi = \sum_{\gamma=1}^{3} \Psi_\gamma}$$

$$\{H_0 + V_\gamma^s(|\vec{x}_\gamma|) + \sum_{\beta=1}^{3} V_\beta^{Coul.}(|\vec{x}_\beta|) - E\}\Psi_\gamma(\vec{x}_\gamma, \vec{y}_\gamma) = -V_\gamma^s(|\vec{x}_\gamma|) \sum_{\beta\neq\gamma} \Psi_\beta(\vec{x}_\beta, \vec{y}_\beta),$$

Partial wave analysis

LS basis $\{(l\lambda)L(\sigma s)S\} \equiv \alpha$

$$W_\alpha(\hat{\mathbf{x}}, \hat{\mathbf{y}}) = <\hat{\mathbf{x}}, \hat{\mathbf{y}}|\alpha> = \left[[Y_l(\hat{\mathbf{x}}) \otimes Y_\lambda(\hat{\mathbf{y}})]^{LL_z} \otimes [\sigma \otimes s]^{SS_z} \right]^{J,J_z}$$

$$\Psi_\gamma(\mathbf{x_i}, \mathbf{y_i}) = \sum_\alpha \frac{\Psi_\gamma^\alpha(x_i, y_i)}{x_i, y_i} W_\alpha(\hat{\mathbf{y}}_\mathbf{i}, \hat{\mathbf{x}}_\mathbf{i})$$

Formalism

$\alpha\Lambda$ spin orbit interaction in LS basis

$$(V^{so}_{\alpha\Lambda})(x) = \frac{2L+1}{2} \sum_{j=l\pm 1/2} (2j+1) \left\{ \begin{array}{ccc} J & L & 1/2 \\ l & j & \lambda \end{array} \right\}^2$$

$$\times (j(j+1) - l(l+1) - 3/4) v^{l}_{so}(x).$$

$\alpha\alpha\Lambda$

The Faddeev equations (two particles are identical)

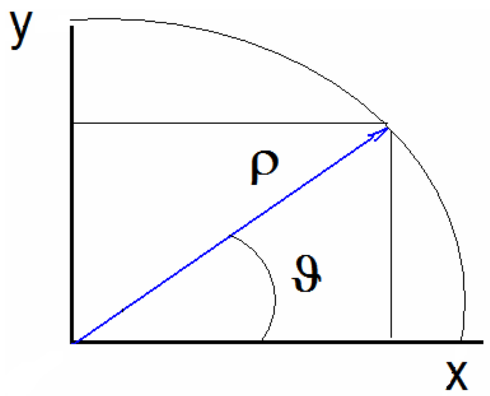
$$(H_0 + V^s_{\alpha\Lambda} + V^{Coul.} - E)W = -V^s_{\alpha\Lambda}(U + P_{12}W),$$
$$(H_0 + V^s_{\alpha\alpha} + V^{Coul.} - E)U = -V^s_{\alpha\alpha}(W + P_{12}W),$$

$$\Psi = U + (1 + P_{12})W$$

Numerical solution of the Faddeev equations

Final equations (after partial analysis) were reduced to a set of 2-dimential equations of (x, y) coordinates.

In polar coordinates ρ and ϑ



$x = \rho \sin(\vartheta)$

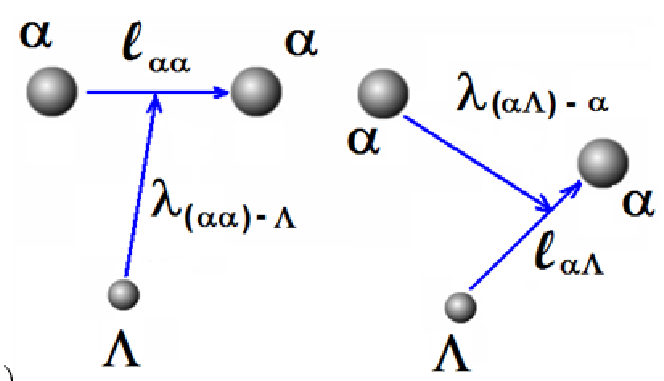
$y = \rho \cos(\vartheta)$

Finite-difference approximation in the hyperraduis and expand the Faddeev components onto **Hermitian cubic splines** with respect to angular variable.

$N_\rho \sim 70$, $N_\vartheta \sim 50$ and the maximum value of the hyperradius $\rho_{max} = 40$ fm.

the method of **inverse iterations** for eigenvalue problem solution

Calculations



Bound states

0^+

$$\begin{pmatrix} (& \{\ell_{\alpha\Lambda}\} &)\,(& \{\ell_{\alpha\alpha}\} &) \\ (& \{\lambda_{(\alpha\Lambda)-\alpha}\} &)\,(& \{\lambda_{(\alpha\alpha)-\Lambda}\} &), \end{pmatrix} \begin{matrix} = (\ 0\ \ 1\ \ 2\ )\ \ (\ 0\ \ 2\ \ 4\ ) \\ = (\ 0\ \ 1\ \ 2\ )\ \ (\ 0\ \ 2\ \ 4\ ) \end{matrix}$$

2^+

$$\begin{pmatrix} (& \{\ell_{\alpha\Lambda}\} &)\,(& \{\ell_{\alpha\alpha}\} &) \\ (& \{\lambda_{(\alpha\Lambda)-\alpha}\} &)\,(& \{\lambda_{(\alpha\alpha)-\Lambda}\} &), \end{pmatrix} \begin{matrix} = (\ 0\ \ 1\ \ 1\ \ 2\ \ 2\ \ 2\ )\ \ (\ 0\ \ 2\ \ 2\ \ 2\ \ 4\ \ 4\ ) \\ = (\ 2\ \ 1\ \ 3\ \ 0\ \ 2\ \ 4\ )\ \ (\ 2\ \ 0\ \ 2\ \ 4\ \ 2\ \ 4\ ) \end{matrix}$$

$\alpha\Lambda$ spin-orbit potential and spin-orbit splitting 2^+ state

$$v^l_{so}(x) = V_o \exp(-\ \alpha x^2)$$

Vo=-9 MeV, α=0.3241 fm^{-2} (Scheerbaum potential)

from Y. Fujiwara, M. Kohno, K. Miyagawa, Y. Suzuki, Phys. Rev. C70 (2004) 047002

Numerical results

Vo=-9 MeV, α=0.3241 fm^{-2}

Bound states

Vo=-9 MeV, α=0.538 fm^{-2}

Binding energy E_B (in MeV), excited energy E_x (in MeV) of degenerated $(3/2^+, 5/2^+)$ spin doublet state of the $\alpha\alpha\Lambda$ system for different models of the $\alpha\alpha$ and $\alpha\Lambda$ interactions. The energy of spin-flip doublet splitting ΔE (in keV) is given for the Scheerbaum (S) potential
Modification S(M) of the spin-orbit potential is made by changing the α parameter to be equal one for corresponding $\alpha\Lambda$ potential. The binding energy is measured from the $\alpha + \alpha + \Lambda$ threshold. Orbital quantum numbers are $\{l_{\alpha\Lambda}\}$=0,1,2, $\{l_{\alpha\alpha}\}$=0,2,4 for the ground state and $\{l_{\alpha\Lambda}\} \leq 2$, $\{l_{\alpha\alpha}\} \leq 4$ for the excited states.

ls pot.		J^π	$\alpha\alpha$	TH	TH(M)	Gibson	MS	Isle	Exp.
	E_B	$\frac{1}{2}^+$	ABa*	-5.990	–	-6.709	-6.663	-8.119	-6.62
			ABd	-5.994	–	-6.751	-6.741	-8.348	
			ABe	-6.280	-6.602	-7.073	-7.080	-8.714	
	E_x	$(ls) = 0$	ABa*	2.562	–	2.621	2.662	2.902	3.04
		$\frac{3}{2}^+$ or $\frac{5}{2}^+$	ABd	2.738	–	2.756	2.773	2.927	
			ABe	2.990	2.99	3.015	3.032	3.193	
S	ΔE	$(\frac{3}{2}^+, \frac{5}{2}^+)$	ABa*	450	–	509	508	511	43
			ABd	346	–	410	447	512	
			ABe	352	378	416	457	525	
S(M)	ΔE	$(\frac{3}{2}^+, \frac{5}{2}^+)$	ABe	126	172	303	–	–	

* $a_0a_2d_4$.

Vo=-54.36 MeV, α=0.538 fm^{-2}

Numerical results

ABe+TH(M)+S(M) model:

	J^π	ABe+TH(M)+S(M)	Y. Fujiwara, et al. (SB+S)	Hiyama et al. (NSC97f)
E_B	$\frac{1}{2}^+$	-6.602	-6.623	–
ΔE	$(\frac{3}{2}^+,\frac{5}{2}^+)$	172	147	160

		ABd+TH(M)+S(MM)
E_x	$\frac{3}{2}^+$	3.016
E_x	$\frac{5}{2}^+$	2.972
ΔE	$(\frac{3}{2}^+,\frac{5}{2}^+)$	44 *

$^*\Delta E^{\text{Exp.}} = 43$ keV

ABe+TH(M)+S(MM) model:

Vo=-2.25 MeV, α=0.538 fm^{-2}

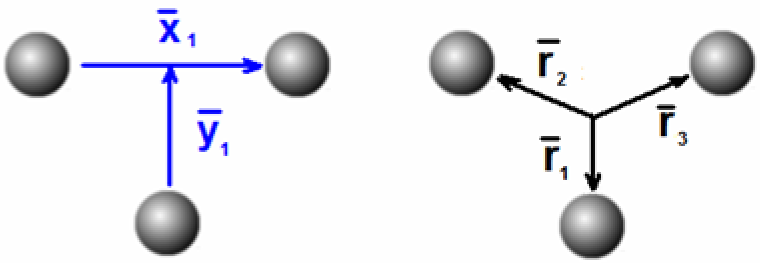
3/2$^+$ is upper member

Resonance states

Method:

Analytical continuation in a parameter (coupling constant) of additional three-body potential

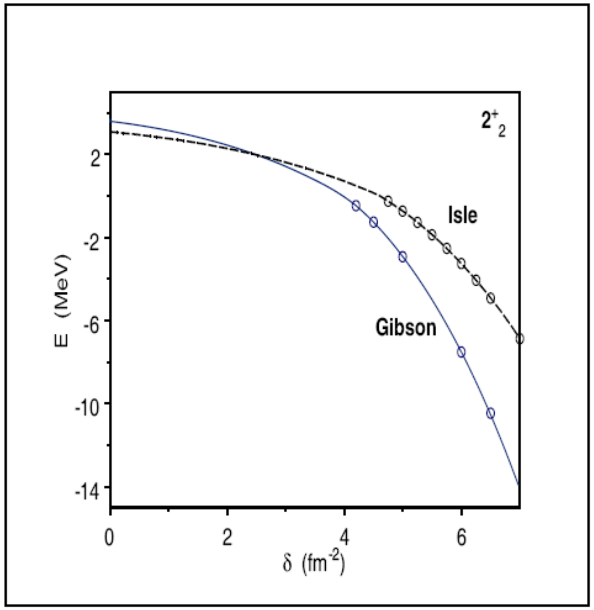
Kukulin V. I., Krasnopol'sky V. M. and Horacek J. Theory of Resonances (Kluwer, Dordrecht) 1989



$$\frac{1}{2}\rho^2 = \sum_{i=1}^{i=3} \bar{\mathbf{r}}_i^2$$

Three-body potential $V_3(\mathrm{x}) = \delta \exp(-\frac{\mathrm{x}^2}{\beta^2}),\qquad \delta \leq 0$

$$\mathrm{x} = \sum_{i=1}^{i=3} \bar{\mathbf{r}}_i^2 \qquad |\delta| \geq |\delta_0|$$

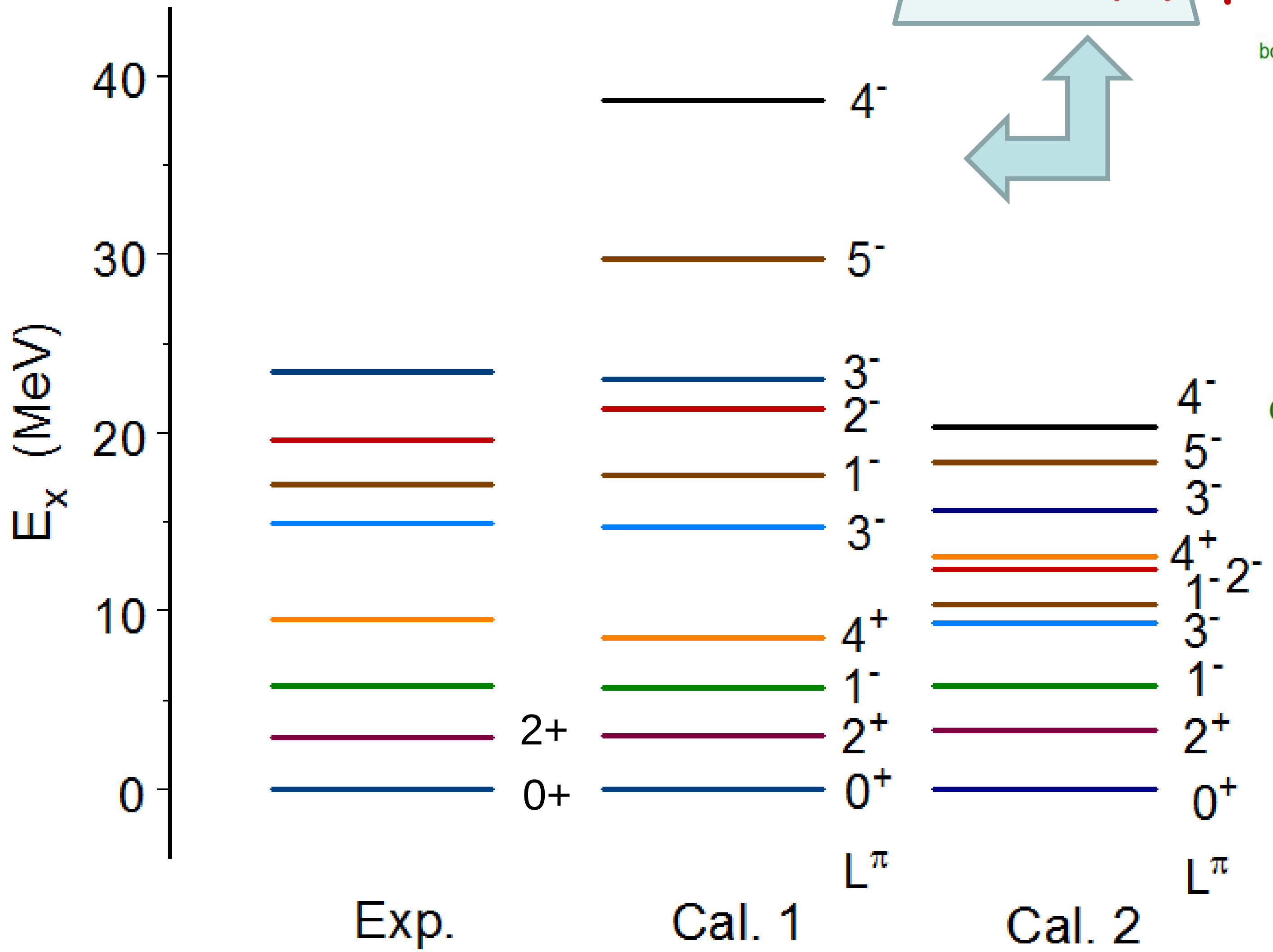


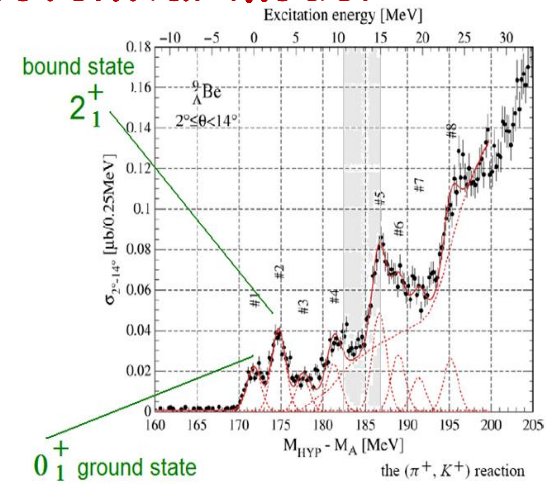
$$\text{Padé approximant:} \quad \sqrt{-E} = \frac{\sum_{i=1}^{N} p_i \xi^i}{1 + \sum_{i=1}^{N} q_i \xi^i}$$

$$\text{where } \xi = \sqrt{\delta_0 - \delta}. \qquad E(\delta = 0) = E_r + i\Gamma/2.$$

Numerical results

Low-lying levels of $\alpha\alpha\Lambda$ system: calculation with the ABe+TH(M) potential model



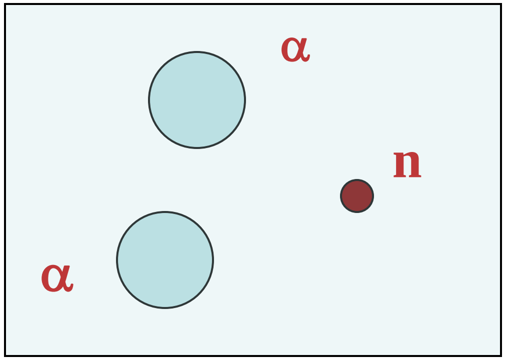


Cal.1 - our calculation

Cal.2 - Yamada, K. Ikeda, H. Bando, Prog. Theor. Phys. 73 (1985) 397

Exp. - experimental data for $(\pi+,K+)$ reaction From “Spectroscopy of Λ hypernuclei” O. Hashimoto, H. Tamura, Progress in Particle and Nuclear Physics, 2006

Cluster model for ^{9}Be

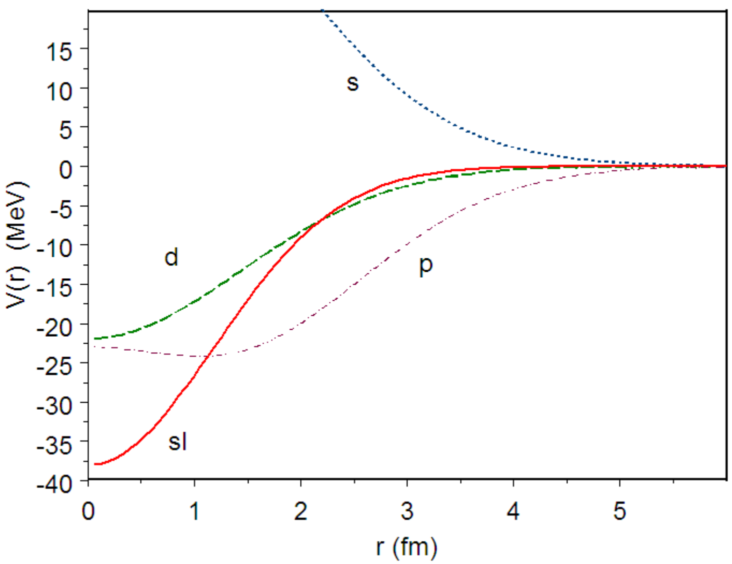


We change Λ to n: $\alpha\alpha n$ system

Potentials

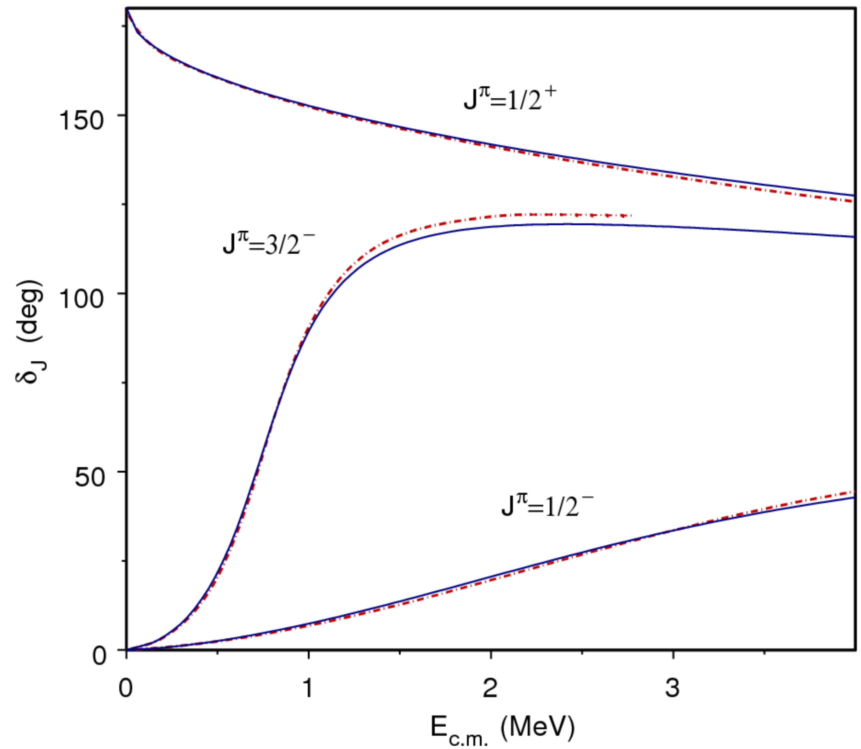
$\alpha\alpha$ -Ali-Bodmer E (Abe)

S. Ali and A. R. Bodmer, Nucl. Phys. 80, 99 (1966)



Partial components of αn -potential

(modified potential of D. Fedorov , A. S. Jensen, 1996(Phys. Lett. B 389))



The αn-phasa shifts.

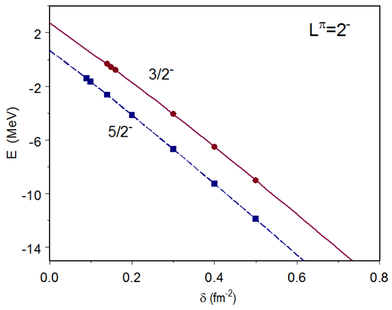
Our calulation (solid line) and Exp. data from Phys. Rev. Lett. 99, 022502 (2007) (dashed line).

Numerical results: bound state

TABLE II. Binding energy E_B of the ^{9}Be ground state ($3/2^-$) in MeV, calculated for various orbital momentum configurations. Energy is measured with respect to the $\alpha + \alpha + n$ threshold. Experimental value is E_B^{ex}=-1.5735 MeV

$\{(l_{\alpha n},\lambda_{(\alpha n)-\alpha})\}$	$\{(l_{\alpha\alpha},\lambda_{(\alpha\alpha)-n})\}$	E_B
(0,1)	(0,1)	unbound
(0,1)(1,0)(1,2)(2,1)		unbound
(0,1)(1,0)(1,2)(2,1)(2,3)		-0.032
(0,1)(1,0)(1,2)(2,1)(2,3)(3,2)		-0.042
(0,1)	(2,1)	unbound
(0,1)(1,0)(1,2)(2,1)(2,3)(3,2)		unbound
	(0,1)(2,1)	-1.403
	(0,1)(2,1)(2,3)	-1.480
	(0,1)(2,1)(2,3)(4,3)	-1.492
	(0,1)(2,1)(2,3)(4,3)(4,5)	-1.493

Numerical results: low-lying resonance states



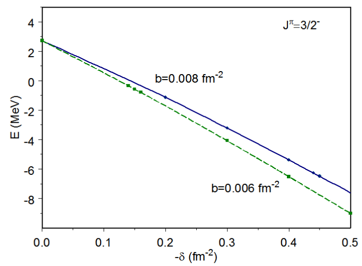
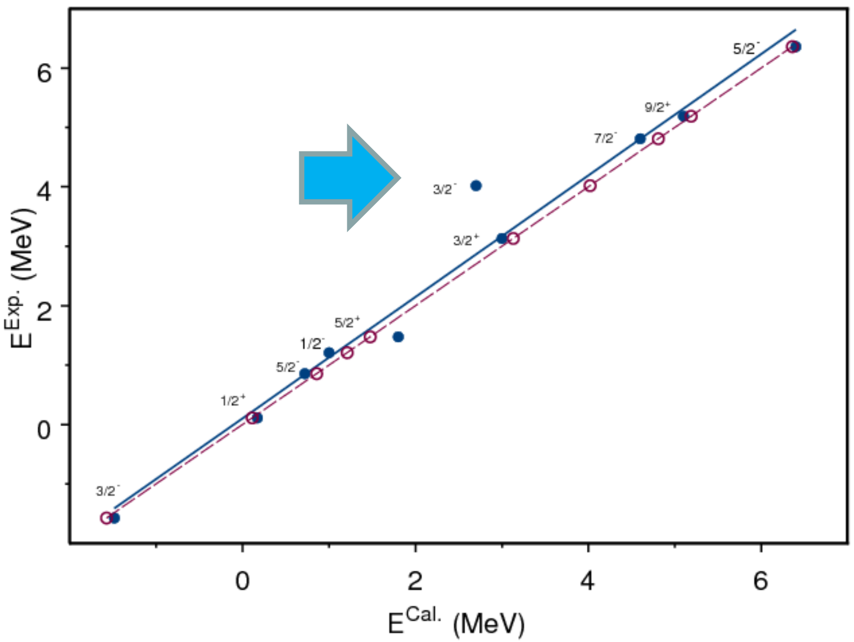


TABLE III. Energy levels in the $\alpha\alpha n$ system and low-lying ^{9}Be spectrum. Results of our calculations are presented in third column. The energy (in MeV) is measured from the $\alpha+\alpha+n$ threshold. Experimental data for ^{9}Be ($T=1/2$) are taken from [35].

L^{π}	J^{π}		[28] (CSM)	[37]	Exp.
0^{+}	$\frac{1}{2}^{+}$	0.3(4)	–	–	0.111±0.007
1^{-}	$\frac{3}{2}^{-}$	-1.493	-2.16	-1.48(4)	-1.5735
	$\frac{1}{2}^{-}$	0.9(0)	1.06	2.24(6)	1.21±0.12
2^{-}	$\frac{5}{2}^{-}$	0.7(2)	0.39	0.73(6)	0.8559±0.0013
	$\frac{3}{2}^{-}$	2.(7)	2.88	–	4.02±0.1
2^{+}	$\frac{5}{2}^{+}$	1.(8)	1.75	–	1.476±0.009
	$\frac{3}{2}^{+}$	3.(0)	3.21	–	3.1312±0.025
3^{-}	$\frac{7}{2}^{-}$	4.(6)	5.02	5.03(6)	4.81±0.06
	$\frac{5}{2}^{-}$	6.(4)	6.57	–	6.36±0.08
4^{+}	$\frac{9}{2}^{+}$	5.(1)	5.04	–	5.19±0.06
	$\frac{7}{2}^{+}$	6.(6)	6.80	–	–
4^{-}	$\frac{9}{2}^{-}$	8.(6)	9.73	8.62(6)	–
	$\frac{7}{2}^{-}$	10.(7)	–	–	–

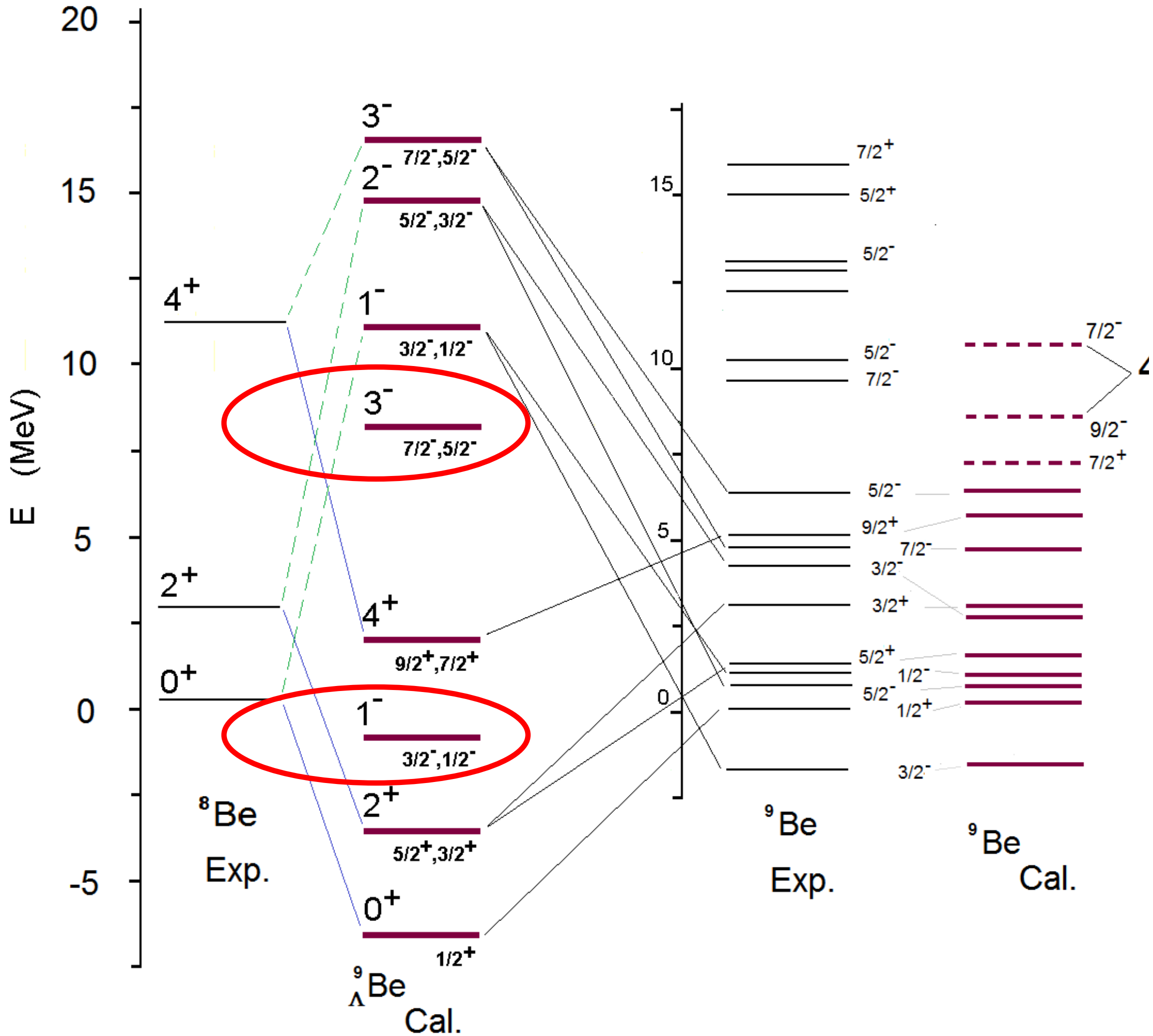


[28] K. Arai, P.Descouvemont, D. Baye, W. N. Catford, Phys.Rev. C 68, 014310 (2003).(cluster model)

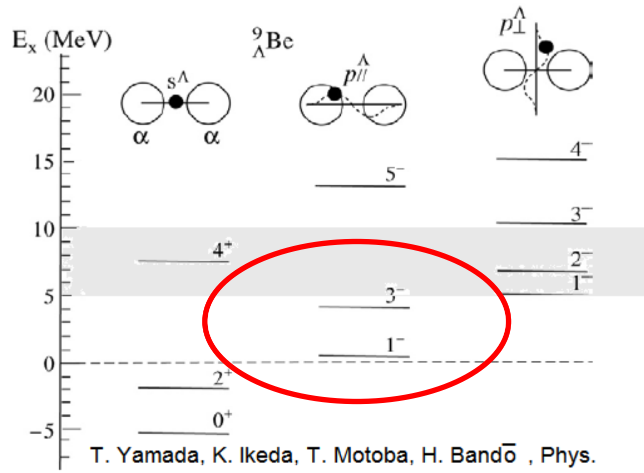
[37] K. M. Nollett et al. Phys. Rev. Lett. 99, 022502 (2007). (Monte Carlo calculations)

Correlation between calculated (Cal.) and experimental (Exp.) spectrum of ^{9}Be (filing dots). Solid line is root mean square fit for the correlation. The dashed line shows the ideal situation when the calculated values coincide with experimental data (open dots). Total momentum of each level is shown.

Numerical results: spectra



O. Hashimoto, H. Tamura / Progress in Particle and Nuclear Physics 2006



T. Yamada, K. Ikeda, T. Motoba, H. Bandō , Phys. Rev. C 38 (1988) 854.

Calculated energy levels of $^{9}_{\Lambda}$Be for three cluster configurations. A band corresponding to genuine hypernuclear states appears as an $\alpha - \alpha - \Lambda(p_{\Lambda})$ configuration.

Calculated (Cal.) and experimental (Exp.) spectrum of ^{9}Be and $^{9}_{\Lambda}$Be. Orbital and total momentum for each level are shown. Energy is measured from $\alpha + \alpha + \Lambda$ and $\alpha + \alpha + n$ thresholds, respectively.

Conclusions

•*The configuration space Faddeev equations are applied to calculate energy states of the $^{9}{}_{\Lambda}$ Be hypernucleus (and ^{9} Be) within three cluster model.*

•*We found the set of phenomenological potentials that reproduces the ground state ½+ binding energy and excitation energy of the 5/2+ and 3/2+ states, simultaneously.*

•*Our calculations reproduce well the experimental data for excitation energies and therefore improve the previous alpha-cluster calculations.*

•*For ^{9}Be we found the set of local phenomenological potentials that reproduces well the ground state binding energy and reasonable -- the energies of low-lying resonances. Also we classified experimental data by total orbital momentum.*

•*It is shown that each energy level of $^{9}{}_{\Lambda}$Be has an analog in ^{9}Be spectrum with the exception of several "genuine hypernuclear states", which agrees qualitatively with previous studies.*

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